Indian Statistical Institute, Bangalore Centre
B.Math (Hons) II Year, Second Semester
Mid-Sem Examination
Optimization

Time: 3 Hours February 27, 2012 Instructor: Pl.Muthuramalingam
Maximum mark you can get is: 35

1. State and prove Lagrange method for a local minima under constraints. [7]

2. Find the shortest distance from the point (a_1, a_2, a_3) in R^3 to the plane whose equation is $b_1x_1+b_2x_2+b_3x_3+b_0 = 0$, by using Lagrange method. [3]

3. Find the point on the line of intersection of the two planes

$$a_1x_1 + a_2x_2 + a_3x_3 + a_0 = 0$$

and

$$b_1x_1 + b_2x_2 + b_3x_3 + b_0 = 0$$

which is nearest to the origin, by using Lagrange method. [7]

4. Let $A : R^n_{col} \to R^k_{col}$ be linear onto map and $\mathbf{b} \varepsilon R^k_{col}$. Here $n > k$. Let $\mathbb{F} = \{\mathbf{x} \varepsilon R^n_{col} : \mathbf{x} \geq 0, A\mathbf{x} = \mathbf{b}\}$ be non empty. Let $A = [\mathbf{a}^1, \mathbf{a}^2, \cdots \mathbf{a}^n]$ be the column representation of A ie $\mathbf{a}^i \varepsilon R^k_{col}$. Assume that $B = \{\mathbf{a}^1, \mathbf{a}^2, \cdots \mathbf{a}^k\}$ is a basis for R^k_{col}. Let $k+1 \leq l \leq n$, and $1 \leq j \leq k$. Let $B(l,j) = B \cup \{\mathbf{a}^l\} \backslash \{\mathbf{a}^j\}$, $\mathbf{a}^l = \sum\limits_1^k r_i \mathbf{a}^i$.

a) Show that $B(l,j)$ is a basis $\langle = \rangle r_j \neq 0$. [2]

Let $B(l,j)$ be a basis. Let $\mathbf{x} = (x_1, x_2, \cdots, x_n)^t$ be a $B-$ basis solution and $\mathbf{y} = (y_1, y_2, \cdots, y_n)^t$ be a $B(l,j)$ basis solution.

b) Find relations between $x_1, x_2, \cdots x_n, y_1, \cdots y_n, r_1, r_2, \cdots r_k$. [1]

c) Show that $x_j = 0 \langle = \rangle \mathbf{x} = \mathbf{y}$ [2]

Assume that $\mathbf{x} \neq \mathbf{y}$. Assume also that $\mathbf{x}$ is (further) a feasible solution.

d) Let $\mathbf{y}$ feasible. Then show that $y_l > 0$ and $x_j > 0$, and $r_j = x_j/y_l$. Let $\theta = \frac{xj}{r_j} = y_l$. [1]

e) Let $\mathbf{y}$ be feasible. Then show that $x_i - \theta r_i \geq 0$ for each $i = 1, 2, \cdots, k$. [1]

f) Let $r_j > 0$ and $T = \{i : 1 \leq i \leq k, r_i > 0\}$ put $\delta = \inf\{\frac{xi}{r_i} : i \varepsilon T\}$. If $\mathbf{y}$ is feasible then show that $\delta = \theta$. [2]

5. Let $F = \left\{ \begin{array}{r} (x_1, x_2) : 3x_1 + 4x_2 \geq 12 \\ 2x_1 + x_2 \geq 4 \\ x_1 \geq 0, x_2 \geq 0 \end{array} \right\}$

a) Let $\mathbf{y} = (a, b)$ where $a < 0$ or $b < 0$. Show that $\mathbf{y}$ is not an extremal direction for F [1]

Hint: Plotting F on a sheet of paper may be helpful.

b) Show that each (a_0, b_0) with $a_0 \geq 0, b_0 \geq 0, a_0^2 + b_0^2 \neq 0$ is an extremal direction for F. [1]

c) Find two linearly independent vectors $\mathbf{y}_1, \mathbf{y}_2$ such that for each $\mathbf{d}$ an extremal direction for F, there exist $\lambda_1, \lambda_2 \geq 0$ such that $\mathbf{d} = \lambda_1 \mathbf{y}_1 + \lambda_2 \mathbf{y}_2$. [1]

d) Convert the inequalities into standard form

$$\{\mathbf{x} \geq \mathbf{0}, A\mathbf{x} = \mathbf{b}, \mathbf{x} \varepsilon R^4_{col}\}.$$

Find all extremal directions for the set $\mathbb{F} = \{\mathbf{x} \geq \mathbf{0}, A\mathbf{x} = \mathbf{b}, \mathbf{x} \varepsilon R^4_{col}\}$.

[1]

e) Find two linearly independent vectors $\mathbf{z}_1, \mathbf{z}_2$ such that for each extremal direction $\mathbf{k}$ for $\mathbb{F}$ we can find $\lambda_1, \lambda_2 \geq 0$ such that $\mathbf{k} = \lambda_1 \mathbf{z}_1 + \lambda_2 \mathbf{z}_2$. [1]

6. Convert the problem:

a) $3x_1 + 4x_2 \leq 12$,

$2x_1 + x_2 \leq 6$,

$x_1 \geq 0, x_2 \geq 0$.

into standard form [1]

b) For the standard form find all the basic solutions. [6]

c) For the standard form find all the basic feasible solutions. [1]